About the bloom structure of closed linear lambda terms

Alain Giorgetti

FEMTO-ST, Univ. Bourgogne Franche-Comté, Besançon, France

Computational Logic and Applications
Versailles, 1-2 July 2019

Context

- Bijection between closed linear lambda terms (CLT) and rooted trivalent maps (RTM)

  [BGJ13] Olivier Bodini, Danièle Gardy, and Alice Jacquot.
  Asymptotics and random sampling for BCI and BCK lambda terms.
  Theoretical Computer Science, 502(0):227 – 238, 2013.

- Towards a formal proof with a proof assistant (Coq)

Summary of previous seasons + talk subject

- *Lambda terms and maps, formally*, at CLA'15
  - With C. Dubois and N. Zeilberger
  - Bijection between labelled CLT and labelled ATM_1
  - Formalized in Coq and Prolog
  - No formal proof there, but validation by enumeration (up to size 4, more than 1000 terms)
- *Lambda terms and maps, formally (II)*, at CLA'18
  - With C. Dubois
  - Bijection splitted into two bijections, with blooms in the middle
  - λ semantics of blooms, visually
  - Map semantics of blooms, visually
  - Formal structural analysis of rooted trivalent maps
  - Prolog specification of blooms
  - First application of blooms: Random and bounded-exhaustive testing
- Today: What are blooms good for?
  - From blooms to CLT: λ semantics of blooms, formally
  - Towards a BCI semantics of blooms

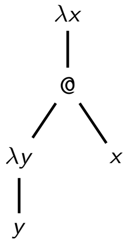
Outline

Closed linear lambda terms (CLT)

- λ terms

$$u ::= \lambda X.\, u \mid u\ u \mid X$$

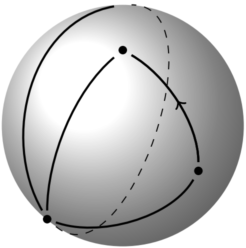
- CLT: closed λ terms where each variable is used exactly once (linearity)
- Example: $\lambda x.\, (\lambda y.\, y)\ x$
- Size: number $n \geq 0$ of applications
- $n + 1$ variables
- $n + 1$ abstractions
- Counted by size (modulo α-equivalence) by the OEIS sequence A062980

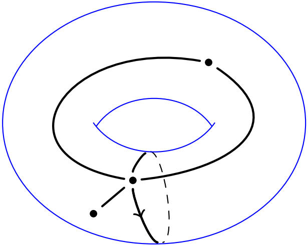


1 5 60 1,105 27,120 828,250 30,220,800 ...

Rooted trivalent maps (RTM)

A (topological) map is a connected graph (loops and multiple edges allowed) drawn on a surface





- Studied (generated, counted, etc) up to isomorphism (orientation-preserving surface diffeomorphism & underlying graph isomorphism) $\rightsquigarrow$ Finitely many distinct maps with a given number of edges
- Rooted: First *dart* (half-edge) distinguished (root) $\rightsquigarrow$ no inner symmetry
- Trivalent: All vertices have degree 3

Bloom structure [Dubois & G.,18]

Definition (Blooms)

A *bloom* b of *size* $n(b)$ (number of internal nodes B_1 and B_2) is either

- B_0 (empty bloom, $n(B_0) = 0$), or
- $(B_1 \_ \, b')$ with a bloom b' such that $n(b') = n(b) - 1$, in $6n(b') + 4$ distinct ways, or
- $(B_2 \; b_1 \; b_2)$ with two blooms b_1 and b_2 such that $n(b) = 1 + n(b_1) + n(b_2)$.

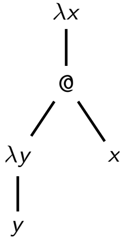
$$b ::= B_0 \mid B_1 \; s \; j \; b \mid B_2 \; b \; b \qquad s ::= \swarrow \mid \searrow \qquad 0 \leq j \leq 3n(b) + 1$$

Also counted by the OEIS sequence A062980

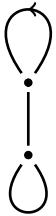
Fine for random and bounded-exhaustive testing
Maybe also for formal reasoning. . .

Blooms in the middle [Dubois & G.,18]

Closed linear λ terms	`bloomlam` ← `lambloom` →	Blooms	`rtmbloom` ← `bloomrtm` →	Rooted trivalent maps



$$(B_1 \searrow 0\ B_0)$$



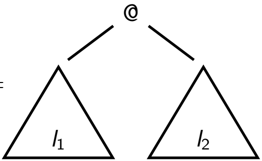
Proposition

Closed linear lambda terms with n applications, blooms with n internal nodes and rooted trivalent maps with $2n$ vertices (and, thus, $3n$ edges) are in bijection.

λ semantics of blooms, visually [Dubois & G.,18]

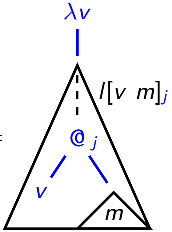
$[\![B_0]\!]_\lambda = \lambda x.x$

$[\![B_2\ b_1\ b_2]\!]_\lambda =$



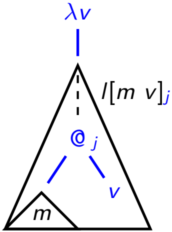
where $[\![b_1]\!]_\lambda = l_1$ and $[\![b_2]\!]_\lambda = l_2$

$[\![B_1\ s\ j\ b]\!]_\lambda =$



if $s = \swarrow$

and



if $s = \searrow$

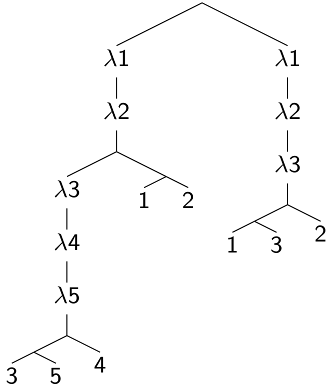
where $l = [\![b]\!]_\lambda$ and $m = l_{|j}$

Outline

Canonical lambda terms

$$u_{m..n} ::= \lambda n.\, u_{m..n+1} \mid u_{m..n}\ u_{m..n} \mid m \mid \ldots \mid n$$

For instance, $u_{1..1}$ generates



λ semantics of blooms: B_0, B_2 and $(B_1\ s\ 0)$

$$b ::= B_0 \mid B_1\ s\ j\ b \mid B_2\ b\ b \qquad s ::= \swarrow \mid \searrow \qquad 0 \leq j \leq 3n(b) + 1$$

Let $[\![b]\!]_{\lambda,n}$ denote the λ semantics of the bloom b, translating it into a canonical λ term with topmost abstractions λn, i.e. generable from $u_{n..n}$

$$[\![B_0]\!]_{\lambda,n} = (\lambda n.\ n) \qquad [\![B_2\ b\ c]\!]_{\lambda,n} = [\![b]\!]_{\lambda,n}\ [\![c]\!]_{\lambda,n}$$

$[\![B_1\ s\ j\ b]\!]_{\lambda,n} = ??$

- depends on s, j and b...
- simpler to define bottom-up, i.e., after having interpreted b
- we define $I(,,,)$ such that $[\![B_1\ s\ j\ b]\!]_{\lambda,n} = I(s, j, n, [\![b]\!]_{\lambda,n+1})$

First case: $j = 0$

$$I(\swarrow, 0, n, u) = (\lambda n.\ n\ u) \qquad I(\searrow, 0, n, u) = (\lambda n.\ u\ n)$$

λ semantics of blooms: application

$[\![b]\!]_{\lambda,n}$ denotes the λ semantics of the bloom b

$$[\![B_1\ s\ j\ b]\!]_{\lambda,n} = l(s,j,n,[\![b]\!]_{\lambda,n+1})$$

Next case: $(B_1\ s\ j\ b)$ when $j \geq 1$ and $[\![b]\!]_{\lambda,n+1} = u\ v$ is an application

Let

- $|w|$ denote the number of nodes in the λ term w
- $\overrightarrow{(\lambda x.u)\ v} =_{\text{def}} u\,[x := v]$ denote one *head reduction* (one step of β-reduction at the top)

First subcase: $1 \leq j \leq |u|$

$$l(s,j,n,u\ v) = \lambda n.\,\overrightarrow{l(s,j-1,n,u)\ n}\ v$$

Second subcase: $|u|+1 \leq j \leq |u|+|v|$

$$l(s,j,n,u\ v) = \lambda n.\,u\ \overrightarrow{l(s,j-(|u|+1),n,v)\ n}$$

λ semantics of blooms: abstraction

$[\![b]\!]_{\lambda,n}$ denotes the λ semantics of the bloom b

$$[\![B_1\ s\ j\ b]\!]_{\lambda,n} = l(s, j, n, [\![b]\!]_{\lambda,n+1})$$

Last case: $(B_1\ s\ j\ b)$ when $j \geq 1$ and $[\![b]\!]_{\lambda,n+1} = \lambda x.\, u$ is an abstraction

Then

$$l(s, j, n, \lambda x.\, u) = \lambda n.\, \lambda x.\, \overrightarrow{l(s, j-1, n, u)\ n}$$

Summary

The λ semantics $[\![b]\!]_{\lambda,n}$ of the bloom b is defined by

$$[\![B_0]\!]_{\lambda,n} = (\lambda n.\, n) \qquad\qquad [\![B_2\ b\ c]\!]_{\lambda,n} = [\![b]\!]_{\lambda,n}\ [\![c]\!]_{\lambda,n}$$

and

$$[\![B_1\ s\ j\ b]\!]_{\lambda,n} = l(s, j, n, [\![b]\!]_{\lambda,n+1})$$

with

$$\begin{array}{rcll}
l(\swarrow, 0, n, u) & = & (\lambda n.\, n\ u) & \\
l(\searrow, 0, n, u) & = & (\lambda n.\, u\ n) & \\
l(s, j, n, u\ v) & = & \lambda n.\, \overrightarrow{l(s, j-1, n, u)\ n}\ v & \text{if } 1 \le j \le |u| \\
l(s, j, n, u\ v) & = & \lambda n.\, u\ \overrightarrow{l(s, j-(|u|+1), n, v)\ n} & \text{if } |u|+1 \le j \\
l(s, j, n, \lambda x.\, u) & = & \lambda n.\, \lambda x.\, \overrightarrow{l(s, j-1, n, u)\ n} & \text{if } j \ge 1
\end{array}$$

where $|w|$ is the number of nodes in the λ term w and $\rightarrow$ denotes one head reduction

Outline

BCI system

BCI terms [H. Curry, *Grundlagen der kombinatorischen Logik*, 1930]

$$d ::= \mathbf{B} \mid \mathbf{C} \mid \mathbf{I} \mid d\ d$$

Let d_λ denote the λ semantics of the BCI term d

$$\begin{array}{rcll} \mathbf{B}_\lambda & = & \lambda x.\, \lambda y.\, \lambda z.\, x\ (y\ z) & \\ \mathbf{C}_\lambda & = & \lambda x.\, \lambda y.\, \lambda z.\, (x\ z)\ y & \\ \mathbf{I}_\lambda & = & \lambda x.\, x & \\ (d\ e)_\lambda & = & d_\lambda\ e_\lambda & \text{for all BCI terms } d \text{ and } e \end{array}$$

Properties

1. d_λ is a CLT
2. Some CLT (e.g., $\lambda y.\, \lambda z.\, y\ z$) cannot be encoded by a BCI term
3. However $CLT = BCI_\lambda^{\rightarrow_\beta^\star}$

$$\begin{array}{rcl} (\mathbf{B}\ \mathbf{I})_\lambda & = & \mathbf{B}_\lambda\ \mathbf{I}_\lambda = (\lambda x.\, \lambda y.\, \lambda z.\, x\ (y\ z))\ (\lambda x.\, x) \\ & & \rightarrow_\beta \lambda y.\, \lambda z.\, (\lambda x.\, x)\ (y\ z) \rightarrow_\beta \lambda y.\, \lambda z.\, y\ z \end{array}$$

4. The same CLT can have several BCI codes, e.g., $((\mathbf{B}\ \mathbf{I})\ \mathbf{I})_\lambda^{\rightarrow_\beta^4} = \mathbf{I}_\lambda$

Bloom semantics of BCI terms

Let d_b denote the bloom semantics of a BCI term d

$$
\begin{aligned}
\mathbf{B}_b &= B_1 \swarrow 2\, (B_1 \swarrow 1\; B_0) \\
\mathbf{C}_b &= B_1 \swarrow 3\, (B_1 \searrow 1\; B_0) \\
\mathbf{I}_b &= B_0 \\
(d\ e)_b &= B_2\ d_b\ e_b \qquad \text{for all BCI terms } d \text{ and } e
\end{aligned}
$$

Proposition (α-equivalent λ semantics)

For all BCI term d,

$$[\![d_b]\!]_{\lambda,n} \equiv_\alpha d_\lambda .$$

Proof by induction on BCI structure

One proof case

$$
\begin{aligned}
\llbracket \mathbf{B}_b \rrbracket_{\lambda,n} &= \llbracket B_1 \ \checkmark\ 2\ (B_1 \ \checkmark\ 1\ B_0) \rrbracket_{\lambda,n} = I(\checkmark, 2, n, \llbracket B_1 \ \checkmark\ 1\ B_0 \rrbracket_{\lambda,n+1}) \\
&= I(\checkmark, 2, n, I(\checkmark, 1, n+1, \llbracket B_0 \rrbracket_{\lambda,n+2})) \\
&= I(\checkmark, 2, n, I(\checkmark, 1, n+1, \lambda(n+2).\,(n+2))) \\
&= I(\checkmark, 2, n, \lambda(n+1).\,\lambda(n+2).\,\overrightarrow{I(\checkmark, 0, n+1, (n+2))\ (n+1)}) \\
&= I(\checkmark, 2, n, \lambda(n+1).\,\lambda(n+2).\,(n+1)\ (n+2)) \\
&= \lambda n.\,\lambda(n+1).\,\overrightarrow{I(\checkmark, 1, n, \lambda(n+2).\,(n+1)\ (n+2))\ n} \\
&= \lambda n.\,\lambda(n+1).\,\overrightarrow{(\lambda n.\,\lambda(n+2).\,\overrightarrow{I(\checkmark, 0, n, (n+1)\ (n+2))\ n})\ n} \\
&= \lambda n.\,\lambda(n+1).\,\overrightarrow{(\lambda n.\,\lambda(n+2).\,n\ ((n+1)\ (n+2)))\ n} \\
&= \lambda n.\,\lambda(n+1).\,\lambda(n+2).\,n\ ((n+1)\ (n+2)) \\
&\equiv_\alpha \lambda x.\,\lambda y.\,\lambda z.\,x\ (y\ z) \\
&= \mathbf{B}_\lambda
\end{aligned}
$$

(+ similar proof for **C**)

Conversely, what should be a BCI semantics for blooms?

Let $[\![b]\!]_{BCI}$ denote any BCI semantics for the bloom b (several solutions)

Expected properties

- *λ-equivalence*

$$([\![b]\!]_{BCI})_\lambda \equiv_{\beta,\alpha} [\![b]\!]_{\lambda,n}$$

composition with λ semantics yields a lambda term (β,α)-equivalent to the λ semantics of blooms

- (λ, r)*-equivalence*: refinement of λ-equivalence to r β-reductions
- *cancellations*

$$[\![b]\!]_{BCI} = d \Leftrightarrow d_b = b \qquad [\![b]\!]_{BCI_b} = b \qquad [\![d_b]\!]_{BCI} = d$$

for all blooms b and BCI terms d

Bloom fragment F_1 for cancellation property

Is there a BCI semantics $[b]_{BCI}$ such that $[b]_{BCI} = d \Leftrightarrow d_b = b$?

- i.e., an inverse of the bloom semantics of BCI terms
- Yes, but it is limited to the sub-family F_1 of blooms b generated by

$$b ::= B_0 \mid B_2\ b\ b \mid B_1 \swarrow 2\ (B_1 \swarrow 1\ B_0) \mid B_1 \swarrow 3\ (B_1 \searrow 1\ B_0)$$

- $[b]_{BCI}$ is defined by

$$[B_0]_{BCI} = \mathbf{I} \qquad [B_2\ b\ c]_{BCI} = [b]_{BCI}\ [c]_{BCI}$$

$$[B_1 \swarrow 2\ (B_1 \swarrow 1\ B_0)]_{BCI} = \mathbf{B} \qquad [B_1 \swarrow 3\ (B_1 \searrow 1\ B_0)]_{BCI} = \mathbf{C}$$

- Bijection between F_1 and BCI terms
- Corollary (also characteristic): $[.]_{BCI}$ satisfies $(\lambda, 0)$-equivalence, i.e., λ-equivalence without beta-reduction

$$([b]_{BCI})_\lambda \equiv_\alpha [\![b]\!]_{\lambda,n}$$

Proofs of $(\lambda, 0)$-equivalence for B_0 and B_2

$$([b]_{BCI})_\lambda \equiv_\alpha^? [\![b]\!]_{\lambda,n}$$

- $[B_0]_{BCI} = \mathbf{I}$

$$([B_0]_{BCI})_\lambda =_{\text{def}} \mathbf{I}_\lambda = (\lambda x. x) \equiv_\alpha (\lambda n. n) = [\![B_0]\!]_{\lambda,n}$$

- $[B_2\ b\ c]_{BCI} = [b]_{BCI}\ [c]_{BCI}$

$$\begin{aligned} ([B_2\ b\ c]_{BCI})_\lambda &= ([b]_{BCI}\ [c]_{BCI})_\lambda = ([b]_{BCI})_\lambda\ ([c]_{BCI})_\lambda \\ &\equiv_\alpha [\![b]\!]_{\lambda,n}\ [\![c]\!]_{\lambda,n} = [\![B_2\ b\ c]\!]_{\lambda,n} \end{aligned}$$

Larger bloom fragment F_2, with β-reduction

Let F_2 be the sub-family of blooms b generated by

$$b ::= B_0 \mid B_2\ b\ b \mid B_1 \swarrow 2\ (B_1 \swarrow 1\ B_0) \mid B_1 \swarrow 3\ (B_1 \searrow 1\ B_0) \mid B_1\ s\ 0\ b$$
$$s ::= \swarrow \mid \searrow$$

Let $[\![b]\!]_{BCI}$ be the BCI semantics defined for blooms b in F_2 by

$$[\![B_0]\!]_{BCI} = \mathbf{I} \qquad [\![B_2\ b\ c]\!]_{BCI} = [\![b]\!]_{BCI}\ [\![c]\!]_{BCI}$$
$$[\![B_1 \swarrow 2\ (B_1 \swarrow 1\ B_0)]\!]_{BCI} = \mathbf{B} \qquad [\![B_1 \swarrow 3\ (B_1 \searrow 1\ B_0)]\!]_{BCI} = \mathbf{C}$$
$$[\![B_1 \swarrow 0\ b]\!]_{BCI} = (\mathbf{C}\ \mathbf{I})\ [\![b]\!]_{BCI} \qquad [\![B_1 \searrow 0\ b]\!]_{BCI} = (\mathbf{B}\ \mathbf{I})\ [\![b]\!]_{BCI}$$

Proposition (λ-equivalence for F_2)

$$\forall b : F_2.\ ([\![b]\!]_{BCI})_\lambda \equiv_{\beta,\alpha} [\![b]\!]_{\lambda,n}$$

Proof by structural induction on F_2

Proof of λ-equivalence for $B_1 \;\swarrow\; 0$

$$([\![b]\!]_{BCI})_\lambda \equiv^?_{\beta,\alpha} [\![b]\!]_{\lambda,n}$$

- Case $[\![B_1 \;\swarrow\; 0\; b]\!]_{BCI} = \mathbf{C}\;\mathbf{I}\;[\![b]\!]_{BCI}$

$$\begin{array}{rcl} [\![B_1 \;\swarrow\; 0\; b]\!]_{\lambda,n} & = & I(\swarrow, 0, n, [\![b]\!]_{\lambda,n+1}) = \lambda n.\, n\; [\![b]\!]_{\lambda,n+1} \\ & \equiv_\beta & (\lambda y.\, \lambda n.\, n\; y)\; [\![b]\!]_{\lambda,n+1} \end{array}$$

Since

$$\begin{array}{rcl} (\mathbf{C}\;\mathbf{I})_\lambda & = & \mathbf{C}_\lambda\; \mathbf{I}_\lambda = (\lambda x.\, \lambda y.\, \lambda z.\, x\; z\; y)\; (\lambda x.\, x) \\ & \equiv_\beta & \lambda y.\, \lambda z.\, (\lambda x.\, x)\; z\; y \\ & \equiv_\beta & \lambda y.\, \lambda z.\, z\; y \end{array}$$

we get

$$\begin{array}{rcl} [\![B_1 \;\swarrow\; 0\; b]\!]_{\lambda,n} & \equiv_{\alpha,\beta} & (\mathbf{C}\;\mathbf{I})_\lambda\; [\![b]\!]_{\lambda,n+1} \equiv_{\alpha,\beta} (\mathbf{C}\;\mathbf{I})_\lambda\; ([\![b]\!]_{BCI})_\lambda \\ & = & ((\mathbf{C}\;\mathbf{I})\; [\![b]\!]_{BCI})_\lambda = ([\![B_1 \;\swarrow\; 0\; b]\!]_{BCI})_\lambda \end{array}$$

- Similar case $[\![B_1 \;\searrow\; 0\; b]\!]_{BCI} = \mathbf{B}\;\mathbf{I}\;[\![b]\!]_{BCI}$

What about cancellation over F_2?

$$[\![b]\!]_{BCI} = d \Leftrightarrow^? d_b = b \qquad [\![b]\!]_{BCI_b} =^? b \qquad [\![d_b]\!]_{BCI} =^? d$$

Small counterexample for $[\![b]\!]_{BCI_b} = b$

$$\begin{aligned} [\![B_1 \swarrow 0\ B_0]\!]_{BCI_b} &= (\mathbf{C\ I\ I})_b = \mathbf{C}_b\ \mathbf{I}_b\ \mathbf{I}_b \\ &= (B_1 \swarrow 3\ (B_1 \searrow 1\ B_0))\ B_0\ B_0 \\ &\neq B_1 \swarrow 0\ B_0 \end{aligned}$$

What about $[\![B_1\ s\ j\ \_]\!]_{BCI}$ for $j \geq 1$?

Blooms:

$$b ::= B_0 \mid B_2\ b\ b \mid B_1\ s\ j\ b \qquad s ::= \swarrow \mid \searrow \qquad 0 \leq j \leq 3n(b)+1$$

Let $[\![b]\!]_{BCI}$ be a BCI semantics of the bloom b, partially defined by

$$[\![B_0]\!]_{BCI} = \mathbf{I} \qquad \text{and} \qquad [\![B_2\ b\ c]\!]_{BCI} = [\![b]\!]_{BCI}\ [\![c]\!]_{BCI}$$

Defining $[\![B_1\ s\ j\ b]\!]_{BCI}$ bottom-up consists in defining $t(,,)$ such that

$$[\![B_1\ s\ j\ b]\!]_{BCI} = t(s, j, [\![b]\!]_{BCI})$$

Can we define $t(,j,)$ by recurrence on j?

$$t(\swarrow, 0, d) = (\mathbf{C}\ \mathbf{I})\ d \qquad t(\searrow, 0, d) = (\mathbf{B}\ \mathbf{I})\ d$$

For $j \geq 1$, $t(s, j, d)$ should be defined at least for all BCI terms generated by

$$d ::= \mathbf{I} \mid d\ d \mid (\mathbf{C}\ \mathbf{I})\ d \mid (\mathbf{B}\ \mathbf{I})\ d$$

$t(s, j, d\ e) = ???$

Issue: application nodes in BCI terms do not correspond to application nodes in their λ semantics $\leadsto$ few chances to ensure λ-equivalence

What about a top-down approach?

$$b ::= B_0 \mid B_2\ b\ b \mid B_1\ s\ j\ b \qquad s ::= \swarrow \mid \searrow \qquad 0 \leq j \leq 3n(b)+1$$

Can we define a BCI semantics $\llbracket b \rrbracket_{BCI}$ of the bloom b top-down?

$$\begin{array}{rcl}
\llbracket B_0 \rrbracket_{BCI} & = & \mathbf{I} \\
\llbracket B_2\ b\ c \rrbracket_{BCI} & = & \llbracket b \rrbracket_{BCI}\ \llbracket c \rrbracket_{BCI} \\
\llbracket B_1\ \swarrow\ 0\ b \rrbracket_{BCI} & = & (\mathbf{C}\ \mathbf{I})\ \llbracket b \rrbracket_{BCI} \\
\llbracket B_1\ \searrow\ 0\ b \rrbracket_{BCI} & = & (\mathbf{B}\ \mathbf{I})\ \llbracket b \rrbracket_{BCI}
\end{array}$$

For $1 \leq j \leq 3n(b)+1$

$$\begin{array}{rcl}
\llbracket B_1\ s\ j\ B_0 \rrbracket_{BCI} & = & ?? \\
\llbracket B_1\ s\ j\ (B_2\ b\ c) \rrbracket_{BCI} & = & ?? \\
\llbracket B_1\ s\ j\ (B_1\ t\ k\ b) \rrbracket_{BCI} & = & ??
\end{array}$$

Basic cases $[\![B_1 \ s \ j \ B_0]\!]_{BCI}$

$s \in \{\swarrow, \searrow\}$ $\qquad$ $n(B_0) = 0$, so $0 \leq j \leq 1$

$$
\begin{aligned}
[\![B_1 \ \swarrow \ 0 \ B_0]\!]_{BCI} &= \mathbf{C\ I\ I} \\
[\![B_1 \ \searrow \ 0 \ B_0]\!]_{BCI} &= \mathbf{B\ I\ I} \\
[\![B_1 \ \swarrow \ 1 \ B_0]\!]_{BCI} &= \mathbf{B\ I} \\
[\![B_1 \ \searrow \ 1 \ B_0]\!]_{BCI} &= \mathbf{C\ I}
\end{aligned}
$$

Binary cases $[\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{BCI}$

$$s \in \{\swarrow, \searrow\} \qquad 1 \le j \le 3n(B_2\ b\ c) + 1 \qquad n(B_2\ b\ c) = 1 + n(b) + n(c)$$

$$\frac{s \in \{\swarrow, \searrow\} \qquad 1 \le j \le 3n(b) + 1}{[\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{BCI} = \mathbf{C}\ [\![B_1\ s\ (j-1)\ b]\!]_{BCI}\ [\![c]\!]_{BCI}}$$

$$\frac{s \in \{\swarrow, \searrow\} \qquad 3n(b) + 2 \le j \le 3n(B_2\ b\ c) + 1}{[\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{BCI} = \mathbf{B}\ [\![b]\!]_{BCI}\ [\![B_1\ s\ (j-(3n(b)+3))\ c]\!]_{BCI}}$$

Proof of λ-equivalence, binary case (1/2)

$$\frac{s \in \{\swarrow, \searrow\} \qquad 1 \leq j \leq 3n(b)+1}{\llbracket B_1\ s\ j\ (B_2\ b\ c)\rrbracket_{BCI} = \mathbf{C}\ \llbracket B_1\ s\ (j-1)\ b\rrbracket_{BCI}\ \llbracket c\rrbracket_{BCI}}$$

$$\begin{array}{rcl}
\llbracket B_1\ s\ j\ (B_2\ b\ c)\rrbracket_{\lambda,n} & = & I(s, j, n, \llbracket B_2\ b\ c\rrbracket_{\lambda,n+1}) \\
& = & I(s, j, n, \llbracket b\rrbracket_{\lambda,n+1}\ \llbracket c\rrbracket_{\lambda,n+1}) \\
& = & \lambda n.\, \overrightarrow{I(s, j-1, n, \llbracket b\rrbracket_{\lambda,n+1})\ n}\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_\beta & \lambda n.\, I(s, j-1, n, \llbracket b\rrbracket_{\lambda,n+1})\ n\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_\beta & \lambda n.\, \llbracket B_1\ s\ (j-1)\ b\rrbracket_{\lambda,n}\ n\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_\beta & (\lambda y.\, \lambda n.\, \llbracket B_1\ s\ (j-1)\ b\rrbracket_{\lambda,n}\ n\ y)\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_\beta & (\lambda x.\, \lambda y.\, \lambda n.\, x\ n\ y)\ \llbracket B_1\ s\ (j-1)\ b\rrbracket_{\lambda,n}\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_{\beta,\alpha} & \mathbf{C}_\lambda\ \llbracket B_1\ s\ (j-1)\ b\rrbracket_{\lambda,n}\ \llbracket c\rrbracket_{\lambda,n+1} \\
& \equiv_{\beta,\alpha} & \mathbf{C}_\lambda\ \llbracket B_1\ s\ (j-1)\ b\rrbracket_{BCI\,\lambda}\ \llbracket c\rrbracket_{BCI\,\lambda} \\
& = & (\mathbf{C}\ \llbracket B_1\ s\ (j-1)\ b\rrbracket_{BCI}\ \llbracket c\rrbracket_{BCI})_\lambda \\
& = & \llbracket B_1\ s\ j\ (B_2\ b\ c)\rrbracket_{BCI\,\lambda}
\end{array}$$

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Proof of λ-equivalence, binary case (2/2)

$$\frac{s \in \{\swarrow, \searrow\} \qquad 3n(b) + 2 \leq j \leq 3n(B_2\ b\ c) + 1}{[\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{BCI} = \mathbf{B}\ [\![b]\!]_{BCI}\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{BCI}}$$

$$\begin{array}{ll}
 & [\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{\lambda,n} \\
= & I(s, j, n, [\![B_2\ b\ c]\!]_{\lambda,n+1}) \\
= & I(s, j, n, [\![b]\!]_{\lambda,n+1}\ [\![c]\!]_{\lambda,n+1}) \\
= & \lambda n.\, [\![b]\!]_{\lambda,n+1}\ \overrightarrow{I(s, j - (|[\![b]\!]_{\lambda,n+1}| + 1), n, [\![c]\!]_{\lambda,n+1})\ n} \\
\equiv_\beta & \lambda n.\, [\![b]\!]_{\lambda,n+1}\ I(s, j - (3n(b)+3), n, [\![c]\!]_{\lambda,n+1})\ n \\
= & \lambda n.\, [\![b]\!]_{\lambda,n+1}\ ([\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{\lambda,n}\ n) \\
\equiv_\beta & (\lambda y.\, \lambda n.\, [\![b]\!]_{\lambda,n+1}\ (y\ n))\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{\lambda,n} \\
\equiv_\beta & (\lambda x.\, \lambda y.\, \lambda n.\, x\ (y\ n))\ [\![b]\!]_{\lambda,n+1}\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{\lambda,n} \\
\equiv_{\beta,\alpha} & \mathbf{B}_\lambda\ [\![b]\!]_{\lambda,n+1}\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{\lambda,n} \\
\equiv_{\beta,\alpha} & \mathbf{B}_\lambda\ [\![b]\!]_{BCI\,\lambda}\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{BCI\,\lambda} \\
= & (\mathbf{B}\ [\![b]\!]_{BCI}\ [\![B_1\ s\ (j - (3n(b)+3))\ c]\!]_{BCI})_\lambda \\
= & [\![B_1\ s\ j\ (B_2\ b\ c)]\!]_{BCI\,\lambda}
\end{array}$$

Unary cases $[\![B_1 \ s \ j \ (B_1 \ t \ k \ b)]\!]_{BCI}$

$$s, t \in \{\swarrow, \searrow\} \qquad 0 \leq k \leq 3n(b) + 1$$

$$1 \leq j \leq |[\![B_1 \ t \ k \ b]\!]_{\lambda,n}| - 1 = 3 + |[\![b]\!]_{\lambda,n}| - 1 = 2 + (3n(b) + 2) = 3n(b) + 4$$

All proofs of λ-equivalence start with

$$\begin{array}{rl} & [\![B_1 \ s \ j \ (B_1 \ t \ k \ b)]\!]_{\lambda,n} \\ = & I(s, j, n, [\![B_1 \ t \ k \ b]\!]_{\lambda,n+1}) \\ = & I(s, j, n, I(t, k, n+1, [\![b]\!]_{\lambda,n+2})) \end{array}$$

Case $k \geq 1$ not yet investigated. For $k = 0$, cases for t and j

$$\begin{array}{rcl} I(\swarrow, 0, n+1, [\![b]\!]_{\lambda,n+2}) & = & \lambda(n+1).\,(n+1) \ [\![b]\!]_{\lambda,n+2} \\ & \rightsquigarrow & \text{cases } j = 1, j = 2 \text{ and } j \geq 3 \\ I(\searrow, 0, n+1, [\![b]\!]_{\lambda,n+2}) & = & \lambda(n+1).\,[\![b]\!]_{\lambda,n+2} \ (n+1) \\ & \rightsquigarrow & \text{cases } j = 1, 2 \leq j \leq 3n(b) + 3 \text{ and } j = 3n(b) + 4 \end{array}$$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \surd$

$$
\begin{array}{rl}
 & [\![B_1\ s\ j\ (B_1\ \surd\ 0\ b)]\!]_{\lambda,n} \\
= & l(s, j, n, \lambda(n+1).\,(n+1)\ [\![b]\!]_{\lambda,n+2}) \\
= & \lambda n.\,\lambda(n+1).\,\overrightarrow{l(s, j-1, n, (n+1)\ [\![b]\!]_{\lambda,n+2})\ n} \\
\equiv_\beta & \lambda n.\,\lambda(n+1).\,l(s, j-1, n, (n+1)\ [\![b]\!]_{\lambda,n+2})\ n
\end{array}
$$

For $j = 1$, cases $s = \surd$ and $s = \backslash$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \surd$, $j = 1$

- $s = \surd$

$$
\begin{array}{rl}
 & [\![B_1\ \surd\ 1\ (B_1\ \surd\ 0\ b)]\!]_{\lambda,n} \\
= & \lambda n.\,\lambda(n+1).\,I(\surd, 0, n, (n+1)\ [\![b]\!]_{\lambda,n+2})\ n \\
= & \lambda n.\,\lambda(n+1).\,(\lambda n.\,n\ ((n+1)\ [\![b]\!]_{\lambda,n+2}))\ n \\
\equiv_\beta & \lambda n.\,\lambda(n+1).\,n\ ((n+1)\ [\![b]\!]_{\lambda,n+2}) \\
\equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,n\ ((n+1)\ x))\ [\![b]\!]_{\lambda,n+2} \\
\equiv_{\beta,\alpha} & (\mathbf{C}\ (\mathbf{B}\ \mathbf{C}\ \mathbf{B}))_\lambda\ [\![b]\!]_{\lambda,n+2} \\
\equiv_{\beta,\alpha} & (\mathbf{C}\ (\mathbf{B}\ \mathbf{C}\ \mathbf{B}))_\lambda\ [\![b]\!]_{BCI}{}_\lambda \\
\equiv_{\beta,\alpha} & (\mathbf{C}\ (\mathbf{B}\ \mathbf{C}\ \mathbf{B})\ [\![b]\!]_{BCI})_\lambda \\
= & [\![B_1\ \surd\ 1\ (B_1\ \surd\ 0\ b)]\!]_{BCI}{}_\lambda
\end{array}
$$

- $s = \searrow$

$$[\![B_1\ \searrow\ 1\ (B_1\ \surd\ 0\ b)]\!]_{BCI} = \mathbf{B}\ \mathbf{C}\ (\mathbf{C}\ \mathbf{I})\ [\![b]\!]_{BCI}$$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \swarrow$, $j = 2$

$$
\begin{array}{ll}
 & [\![B_1\ s\ 2\ (B_1\ \swarrow\ 0\ b)]\!]_{\lambda,n} \\
= & \lambda n.\,\lambda(n+1).\,l(s, 1, n, (n+1)\ [\![b]\!]_{\lambda,n+2})\ n \\
= & \lambda n.\,\lambda(n+1).\,(\lambda n.\,\overrightarrow{l(s, 0, n, n+1)\ n}\ [\![b]\!]_{\lambda,n+2}))\ n \\
= & \lambda n.\,\lambda(n+1).\,\overrightarrow{l(s, 0, n, n+1)\ n}\ [\![b]\!]_{\lambda,n+2}) \\
\equiv_\beta & \lambda n.\,\lambda(n+1).\,l(s, 0, n, n+1)\ n)\ [\![b]\!]_{\lambda,n+2})
\end{array}
$$

- $s = \swarrow$

$$
\begin{array}{ll}
= & \lambda n.\,\lambda(n+1).\,((\lambda n.\,n\ (n+1))\ n)\ [\![b]\!]_{\lambda,n+2}) \\
\equiv_\beta & \lambda n.\,\lambda(n+1).\,(n\ (n+1))\ [\![b]\!]_{\lambda,n+2}) \\
\equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,n\ (n+1)\ x)\ [\![b]\!]_{\lambda,n+2} \\
\equiv_\beta & (\mathbf{C}\ \mathbf{C})_\lambda\ [\![b]\!]_{BCI\,\lambda} \\
= & (\mathbf{C}\ \mathbf{C}\ [\![b]\!]_{BCI})_\lambda
\end{array}
$$

- $s = \searrow$

$$
\begin{array}{ll}
\equiv_\beta & \lambda n.\,\lambda(n+1).\,((n+1)\ n)\ [\![b]\!]_{\lambda,n+2}) \\
\equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,(n+1)\ n\ x)\ [\![b]\!]_{\lambda,n+2} \\
\equiv_\beta & (\mathbf{B}\ \mathbf{C}\ (\mathbf{C}\ \mathbf{C}))_\lambda\ [\![b]\!]_{BCI\,\lambda}
\end{array}
$$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \surd$, $j \geq 3$

$$
\begin{array}{cl}
 & [\![B_1\ s\ j\ (B_1\ \surd\ 0\ b)]\!]_{\lambda,n} \\
= & \lambda n.\,\lambda(n+1).\,l(s, j-1, n, (n+1)\ [\![b]\!]_{\lambda,n+2})\ n \\
= & \lambda n.\,\lambda(n+1).\,(\lambda n.\,(n+1)\ \overrightarrow{l(s, j-1-(|(n+1)|+1), n, [\![b]\!]_{\lambda,n+2})\ n})\ n \\
\equiv_\beta & \lambda n.\,\lambda(n+1).\,(n+1)\ (l(s, j-3, n, [\![b]\!]_{\lambda,n+2})\ n) \\
\equiv_\alpha & \lambda n.\,\lambda(n+1).\,(n+1)\ ([\![B_1\ s\ (j-3)\ b]\!]_{\lambda,n+1}\ n) \\
\equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,(n+1)\ (x\ n))\ [\![B_1\ s\ (j-3)\ b]\!]_{\lambda,n+1} \\
\equiv_{\beta,\alpha} & (\mathbf{B}\ (\mathbf{C}\ \mathbf{I}))_\lambda\ [\![B_1\ s\ (j-3)\ b]\!]_{BCI\,\lambda} \\
= & (\mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ [\![B_1\ s\ (j-3)\ b]\!]_{BCI})_\lambda \\
= & [\![B_1\ s\ j\ (B_1\ \surd\ 0\ b)]\!]_{BCI\,\lambda}
\end{array}
$$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \searrow$

$$
\begin{array}{ll}
 & [\![B_1\ s\ j\ (B_1\ \searrow\ 0\ b)]\!]_{\lambda,n} \\
= & l(s, j, n, \lambda(n+1).\, [\![b]\!]_{\lambda,n+2}\ (n+1)) \\
= & \lambda n.\, \lambda(n+1).\, \overrightarrow{l(s, j-1, n, [\![b]\!]_{\lambda,n+2}\ (n+1))\ n} \\
= & \lambda n.\, \lambda(n+1).\, l(s, j-1, n, [\![b]\!]_{\lambda,n+2}\ (n+1))\ n
\end{array}
$$

For $j = 1$

- $s = \swarrow$

$$
\begin{array}{ll}
= & \lambda n.\, \lambda(n+1).\, l(\swarrow, 0, n, [\![b]\!]_{\lambda,n+2}\ (n+1))\ n \\
= & \lambda n.\, \lambda(n+1).\, n\ ([\![b]\!]_{\lambda,n+2}\ (n+1)) \\
\equiv_\beta & (\lambda x.\, \lambda n.\, \lambda(n+1).\, n\ (x\ (n+1)))\ [\![b]\!]_{\lambda,n+2} \\
\equiv_{\beta,\alpha} & (\mathbf{C}\ \mathbf{B})_\lambda\ [\![b]\!]_{BCI\,\lambda} \\
= & (\mathbf{C}\ \mathbf{B}\ [\![b]\!]_{BCI})_\lambda
\end{array}
$$

- $s = \searrow$

$$
\begin{array}{ll}
= & \lambda n.\, \lambda(n+1).\, l(\searrow, 0, n, [\![b]\!]_{\lambda,n+2}\ (n+1))\ n \\
= & \lambda n.\, \lambda(n+1).\, ([\![b]\!]_{\lambda,n+2}\ (n+1))\ n \\
\equiv_\beta & (\lambda x.\, \lambda n.\, \lambda(n+1).\, (x\ (n+1))\ n)\ [\![b]\!]_{\lambda,n+2} \\
\equiv_{\beta,\alpha} & \mathbf{C}_\lambda\ [\![b]\!]_{BCI\,\lambda}
\end{array}
$$

$\llbracket B_1\ s\ j\ (B_1\ t\ k\ b)\rrbracket_{BCI},\ k = 0,\ t = \searrow,\ 2 \le j \le 3n(b) + 3$

$$1 \le j - 1 \le 3n(b) + 2 = |\llbracket b \rrbracket_{\lambda, n+2}|$$

$$\begin{array}{ll}
 & \llbracket B_1\ s\ j\ (B_1\ \searrow\ 0\ b)\rrbracket_{\lambda,n} \\
= & \lambda n.\, \lambda(n+1).\, l(s, j-1, n, \llbracket b \rrbracket_{\lambda,n+2}\ (n+1))\ n \\
= & \lambda n.\, \lambda(n+1).\, (\lambda n.\, \overrightarrow{l(s, j-2, n, \llbracket b \rrbracket_{\lambda,n+2})\ n}\ (n+1))\ n \\
\equiv_\beta & \lambda n.\, \lambda(n+1).\, (l(s, j-2, n, \llbracket b \rrbracket_{\lambda,n+2})\ n)\ (n+1) \\
\equiv_\beta & \lambda n.\, \lambda(n+1).\, (\llbracket B_1\ s\ (j-2)\ b \rrbracket_{\lambda,n+2}\ n)\ (n+1) \\
\equiv_\beta & (\lambda x.\, \lambda n.\, \lambda(n+1).\, (x\ n)\ (n+1))\ \llbracket B_1\ s\ (j-2)\ b \rrbracket_{\lambda,n+2} \\
\equiv_{\beta,\alpha} & (\mathbf{B\ C\ C})_\lambda\ {\llbracket B_1\ s\ (j-2)\ b \rrbracket_{BCI}}_\lambda \\
= & (\mathbf{B\ C\ C}\ \llbracket B_1\ s\ (j-2)\ b \rrbracket_{BCI})_\lambda
\end{array}$$

$[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$, $k = 0$, $t = \searrow$, $j = 3n(b) + 4$

$$j - 1 = 3n(b) + 3$$

$$\begin{array}{rl} & [\![B_1\ s\ (3n(b)+4)\ (B_1\ \searrow\ 0\ b)]\!]_{\lambda,n} \\ = & \lambda n.\,\lambda(n+1).\,(\lambda n.\,[\![b]\!]_{\lambda,n+2}\ \overrightarrow{I(s, 3n(b)+3-(|[\![b]\!]_{\lambda,n+2}|+1), n, (n+1))\ n})\ n \\ \equiv_\beta & \lambda n.\,\lambda(n+1).\,[\![b]\!]_{\lambda,n+2}\ \overrightarrow{I(s, 0, n, (n+1))\ n} \end{array}$$

- $s = \swarrow$

$$\begin{array}{rl} \equiv_\beta & \lambda n.\,\lambda(n+1).\,[\![b]\!]_{\lambda,n+2}\ (n\ (n+1)) \\ \equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,x\ (n\ (n+1)))\ [\![b]\!]_{\lambda,n+2} \\ \equiv_{\beta,\alpha} & \mathbf{B}_\lambda\ [\![b]\!]_{BCI\,\lambda} \\ = & (\mathbf{B}\ [\![b]\!]_{BCI})_\lambda \end{array}$$

- $s = \searrow$

$$\begin{array}{rl} \equiv_\beta & \lambda n.\,\lambda(n+1).\,[\![b]\!]_{\lambda,n+2}\ ((n+1)\ n) \\ \equiv_\beta & (\lambda x.\,\lambda n.\,\lambda(n+1).\,x\ ((n+1)\ n))\ [\![b]\!]_{\lambda,n+2} \\ \equiv_{\beta,\alpha} & (\mathbf{B}\ \mathbf{C}\ \mathbf{B})_\lambda\ [\![b]\!]_{BCI\,\lambda} \\ \equiv_{\beta,\alpha} & (\mathbf{B}\ \mathbf{C}\ \mathbf{B}\ [\![b]\!]_{BCI})_\lambda \end{array}$$

Summary of unary cases $[\![B_1\ s\ j\ (B_1\ t\ k\ b)]\!]_{BCI}$

$$s, t \in \{\swarrow, \searrow\} \qquad 0 \leq k \leq 3n(b)+1 \qquad 1 \leq j \leq 3n(b)+4$$

For $k = 0$:

$$\begin{array}{rcll}
[\![B_1\ \swarrow\ 1\ (B_1\ \swarrow\ 0\ b)]\!]_{BCI} & = & \mathbf{C}\ (\mathbf{B}\ \mathbf{C}\ \mathbf{B})\ [\![b]\!]_{BCI} & \\
[\![B_1\ \searrow\ 1\ (B_1\ \swarrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ \mathbf{C}\ (\mathbf{C}\ \mathbf{I})\ [\![b]\!]_{BCI} & \\
[\![B_1\ \swarrow\ 2\ (B_1\ \swarrow\ 0\ b)]\!]_{BCI} & = & \mathbf{C}\ \mathbf{C}\ [\![b]\!]_{BCI} & \\
[\![B_1\ \searrow\ 2\ (B_1\ \swarrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ \mathbf{C}\ (\mathbf{C}\ \mathbf{C})\ [\![b]\!]_{BCI} & \\
[\![B_1\ s\ j\ (B_1\ \swarrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ (\mathbf{C}\ \mathbf{I})\ [\![B_1\ s\ (j-3)\ b]\!]_{BCI} & \text{if } 3 \leq j \\
 & & & \\
[\![B_1\ \swarrow\ 1\ (B_1\ \searrow\ 0\ b)]\!]_{BCI} & = & \mathbf{C}\ \mathbf{B}\ [\![b]\!]_{BCI} & \\
[\![B_1\ \searrow\ 1\ (B_1\ \searrow\ 0\ b)]\!]_{BCI} & = & \mathbf{C}\ [\![b]\!]_{BCI} & \\
[\![B_1\ s\ j\ (B_1\ \searrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ \mathbf{C}\ \mathbf{C}\ [\![B_1\ s\ (j-2)\ b]\!]_{BCI} & \text{if } 2 \leq j \leq 3n(b)+3 \\
[\![B_1\ \swarrow\ j\ (B_1\ \searrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ [\![b]\!]_{BCI} & \text{if } j = 3n(b)+4 \\
[\![B_1\ \searrow\ j\ (B_1\ \searrow\ 0\ b)]\!]_{BCI} & = & \mathbf{B}\ \mathbf{C}\ \mathbf{B}\ [\![b]\!]_{BCI} & \text{if } j = 3n(b)+4
\end{array}$$

Third bloom fragment

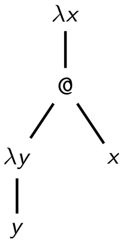
With $k = 0$ only, the former semantics covers the fragment F_3 of blooms generated by

$$\begin{array}{rcl} b & ::= & B_0 \mid B_2\ b\ b \mid B_1\ s\ 0\ b \mid B_1\ s\ j\ B_0 \mid B_1\ s\ j\ (B_2\ b\ c) \mid \\ & & B_1\ s\ j\ (B_1\ t\ 0\ b) \\ s, t & ::= & \swarrow \mid \searrow \\ j & \geq & 1 \end{array}$$

Outline

Four points of view

λ terms — Blooms — RTM

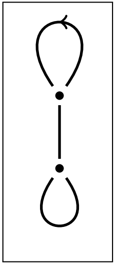


bloomlam ←

lambloom →

$b_1(\setminus, i, b_0)$

bloomrtm ↔



↑ ↓[1]

BCI terms

[1] not yet complete

Conclusion

- Work in progress
- Partially assisted by Prolog code (semantics, generation of correspondence tables)

Numerous perspectives

- Combinatorial study of bloom fragments for BCI semantics, and their BCI and λ counterparts
- Refinement by number of β-reductions
- Meaning for rooted trivalent maps
- More work for formal proof

- Thanks for your attention
- Questions?