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ALMOST ALL CLASSICAL THEOREMS ARE INTUITIONISTIC

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Abstract. Canonical expressions represent the implicative propositions (i.e., the propositions with only implications) up-to renaming of variables. Using a Monte-Carlo approach, we explore the model of canonical expressions in order to confirm the paradox that says that *asymptotically almost all classical theorems are intuitionistic*. Actually we found that more than 96.6% of classical theorems are intuitionistic among propositions of size 100.

Résumé. Les expressions canoniques représentent les propositions implicatives (autrement dit, les propositions ne contenant que des implications) à un renommage des variables près. En utilisant une approche à la Monte-Carlo, j'explore le modèle des expressions canoniques dans le but de confirmer le paradoxe qui dit qu'*asymptotiquement presque tous les théorèmes classiques sont intuitionnistes*. En fait, je prouve que plus de 96, 6% des théorèmes classiques sont intuitionnistes parmi les propositions de taille 100.

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In 2007, Marek Zaionc coauthored two papers [1,2], corresponding to two models of the calculus of implicative propositions and presenting the following paradox, namely that *asymptotically almost all classical theorems are intuitionistic*, which we call, in short, *Zaionc paradox*. This says that when the size of the propositions grows, the ratio of the number of intuitionistic theorems over the number of classical theorems goes up to one.

Usually people believe that there is a gap between between intuitionistic and classical theorems. For instance, if you interview mathematicians whether there

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are much more classical theorems that intuitionistic ones, they would say that yes, classical theorems are more than intuitionistic ones and that if you pick at random big theorems, you will easily find one which is classical not intuitionistic. This difference is important, because simple methods for checking membership to one class or the other in the calculus of propositions are different, in one case providing a proof (or a Kripke model) and in the other case exhibiting a boolean assignment. Perhaps the present contribution starts a new discipline, which I propose to call *experimental logic*.

In the current paper, we focus on the model of [2], which we call *canonical expressions*. They have been introduced by Genitrini, Kozik and Zaionc [2] and more recently by Tarau and de Paiva [3, 4]. A canonical expression is a representative of a class of implicative propositions (propositions that contain only implication $\to$) which differ only by the name assigned to the variables. Whereas Genitrini, Kozik and Zaionc addressed the mathematical aspect of this model, Tarau and de Paiva tried to explicitly generate all the canonical expressions of a given size and faced up to combinatorial explosion, because canonical expressions grow super exponentially in size when the number on variables increases. In this paper, I check experimentally Zaionc paradox, adopting a Monte-Carlo approach to observe how this paradox emerges. Indeed I designed a linear algorithm to randomly generate canonical expressions. Therefore I can consider large samples of random (for a uniform distribution) canonical expressions and count how many canonical expressions in that samples are intuitionistic theorems or classical theorems. The experiments, centered around canonical expressions of size 100, show that the numbers we get for both sets (classical and intuitionistic) are very close confirming experimentally the paradox, with a ratio 96.6% much better than this obtained on the model of Genitrini et al. which yields 36% for canonical expressions of size 100. As a by product we obtain programs generating large random canonical expressions, large random intuitionistic theorems or large random classical theorems.

The programs used in this paper can be found on GitHub. The full paper can be found here

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