

Closable and uniquely closable skeletons of untyped lambda terms, formally*

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In 2017, Olivier Bodini and Paul Tarau [BT17] introduced closable and uniquely closable Motzkin trees as “skeletons” to describe the structure of closed untyped lambda terms. They defined efficient Prolog generators for these skeletons and studied their statistical properties.

We present a formalization of these different notions in the Coq proof assistant [BC04, Coq21], with formal proofs of equivalence between several definitions that underlie the generators designed by Bodini and Tarau. We also present random generators to be used with QuickChick [LP18], and extend the discourse to open λ -terms.

To facilitate this work and generalize the approach, we propose generic tools to help setting up the correspondence between two isomorphic types more easily. We hope such a methodology could be reused to deal with other families of objects, having different and isomorphic representations.

References

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