ϱ-property of B-terms*

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* B-term is a combinator built from $B = \lambda xyz.x(yz)$ alone.

ϱ-property of combinators

Definition of $(X)_n$ (self right application of n X's).

Let X be a combinator.

$(X)_n \overset{\text{def}}{=} X\,X\,X \ldots X$ *where application is left-associative*

Formally, $(X)_1 = X$ *and* $(X)_{n+1} = (X)_n\,X$ *for* $n \geq 1$.

Definition (ϱ-property).

A combinator X has ϱ-property if $\{(X)_n \mid n \in \mathbb{N}\}$ *is finite up to $\beta\eta$-equivalence.*

Example (ϱ-property of B and $B\,B$)

$(B)_{10} =_{\beta\eta} (B)_6$ (which implies $(B)_{10+k} =_{\beta\eta} (B)_{6+k}$ for all $k \geq 0$)

$(B\,B)_{52} =_{\beta\eta} (B\,B)_{32}$

Conjecture on ϱ-property

Nakano conjectured in 2008 that:

B-term X has ϱ-property iff $X =_{\beta\eta} B^n B$ for some $n \geq 0$

where $B^0 B = B$ and $B^{k+1} B = B (B^k B)$ for $k \geq 0$.

(if part) Known only for $n \leq 6$ by computation.

$(B^0 B)_6 =_{\beta\eta} (B^0 B)_{10}$ $(B^1 B)_{32} =_{\beta\eta} (B^1 B)_{52}$ $(B^2 B)_{294} =_{\beta\eta} (B^2 B)_{258}$

$(B^3 B)_{10036} =_{\beta\eta} (B^3 B)_{4240}$ $(B^4 B)_{622659} =_{\beta\eta} (B^4 B)_{191206}$

$(B^5 B)_{1000685878} =_{\beta\eta} (B^5 B)_{766241307}$

$(B^6 B)_{2980054085040} =_{\beta\eta} (B^6 B)_{2641033883877}$

Visit `https://github.com/ksk/Rho` for the implementation.

(only if part) Known only for some special forms of B-terms.

See [Ikebuchi&N. 2020] for the detail.

Reference

[Ikebuchi&N. 2020]
Mirai Ikebuchi and Keisuke Nakano
"On Properties of B-terms"
Logical Methods in Computer Science, 16(2), 2020.

where a motivational story for the problem is included.