Algebraic and combinatorial structures intrinsic to functional Programming

by
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Talk proposal:

Lambda Calculus is the starting point of all functional programming. Since Church ([6],[22]) noticed the undecidablity of the word problem for semigroups,it has been understood that certain algebraic strutures are embedded in lambda calculus and related systems such as combinatory logic ([7]), simply typed lambda calculus with surjective pairing ([17]), Godel's theory T from proof theory ([16]), and John Backus' programming system FP ([3],[24]). One nice aspect of algebra is that it leads to generators and generators lead to enumeration.

The algebraic structures include the "positive parts" of the Freyd, Heller,Thompson, Higman groups F, T and V ([26],[4],[21]),the free Cartesian monoid (of which V is "the" group,[25])),the profinte group of hereditary permutations ([8]),and the near semirings and b.a.d. algebras of [21]. Here,the connection to combinatorics begins with the observation that the group F is a subgroup of the automorphism group of the ordered rooted binary trees ([5]). In addition,the B,I monoid induces a monoid structure on lattice paths from (0,0) that end at, but never go above, the diagonal ([15]).The correspondence between such paths and trees,ballot sequences, polygonal triangulations, and parenthesized products is well know from elementary combinatorics. There are some amusing consequences for the theory of partitions of integers as well([2]) .

In this talk we would like to give a somewhat eccentric survey of our work in this area.

References:

[1]Andrews, G. Theory of partitions
Encyclopedia of Mathematics Vol 2
Addison-Welsley 1976

[2] personal communication 1988

[3]Backus,J. Can programming be liberated from the von Neumann style?
Communications of the ACM,
Vol 21, No 8, August 1978

[4]Birget J-C. Monoid generalization of the Richard Thompsom groups
Journal of Pure and Applied Algebra
13, 2009 pps 264-278

[5]Cannon,Floyd,& Parry Introductory remarks on Richard Thompson's groups

L' Enseignement Mathematique 42
(1996) pps 2215-256

[6]Church,Alonzo
A Note on the Entscheidungsproblem
J. of Symbolic Logic, vol. 1 1936

[7]Curry, Haskell B., Feys, Robert
Combinatory Logic. Vol. I
North Holland 1958

[8]Dazani-Ciancaglini, Mariangiola
Characterization of normal forms ...
T.C.S. 2, 1976

[9]Freyd P. personal communication 1988

[10]Freyd P. & Heller A. Spliting homotopy invariants II
Journal of Pure and Applied Algebra
89,1993.pps 93-195

[11]Gray A. & Pardue K. Products in a category with one object
https://arxiv.org/abs/1604.03999

[12]Jonsson B. & Tarski A. On two properties of free algebras
Math. Scand. 9, 1961, pps 95-101

[13]Lambek J. From lambda calculus to Cartesian closed categories
To H.B. Curry; Essays in Combinatory Logic...
Seldin J. & Hindley R. eds, Academic Press 1980

[14]Smirnov D.M. Cantor algebras with one generator
Algebra and Logic 10, 1971, pps 40-49

[15]Statman R.
Combinators and the theory of partitions,
CMU Research Report No. 88-31 1988.

[16] Freyd's Hierarchy of Combinator Monoids
Proceedings of 1991 Logic in Computer Science Conference
I.E.E.E., (July, 1991).

[17] Simply typed lambda calculus with surjective
pairing,
CMU Department of Mathematics Research Report
92-164, 1992

[18] On Cartesian monoids,
Proceedings of Computer Science Logic 1996.
Springer Lecture Notes in Computer Sciences
1258, Springer-Verlag, 1996, pps 446-459

[19] Surjective pairing revisited
Liber Amicorum for Roel deVrijer,

Klop J.W., van Oostrom J., van Raamsdonk F. eds,
University of Amsterdam, 2009

[20] Cartesian monoids
Electronic Notes in Theoretical Computer Science
265, Elsevier 2010 pps 437-451

[21] Near semi-rings and lambda calculus
TLCA 2015

[22] On the representation of semigroups and other
congruences in the lambda calculus
CSL 2016

[23] Representation of Cartesian monoids in and around
the piecewise shift operators
to appear in
Mathematical Structures in Computer Science

[24] Minimal Backus FP is Turing complete
Higher Dimensional Rewriting and Algebra
July 7 2018, Oxford

[25] Products in a category with only one object
ACT 2020

[26]Thompson R., Embeddings into finitely generated simple groups
which preserve the word problem
in Adian S.,Boone W. & Higman G. (eds)
Word Problems

[27]van Hoorn & van Rootselaar, Fundamental notions in the theory of
semi-near rings
Comp. Math. 18 1967 pps 65-78