Locally finite logics have the density

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Let $Form$ be the set of propositional formulas in a given language and $A \subset Form$. The density of A, denoted by $\mu(A)$, is defined as follows:

$$\mu(A) = \lim_{n\to\infty} \frac{|\{\alpha \in A : \|\alpha\| = n\}|}{|\{\alpha \in Form : \|\alpha\| = n\}|}.$$

The length $\|\alpha\|$ of a formula α is defined in the standard way and $|A|$ denotes the cardinality of A. If A is the set of tautologies of a given logic, then $\mu(A)$ is called *the density of truth* of this logic. Note that $\mu(A)$ does not exist for some sets (or logics) A.
The density of truth was wildly investigated in literature. First, the density was computed for various fragments of classical propositional logic. Second, M.Moczurad, J. Tyszkiewicz and M. Zaionc [5] raised the question if classical and intuitionistic logics have the same density. Some partial results are contained, for example, in [4] and [2].
Our aim is to give *more general conditions on the existence of the density of truth*. The notion of local finiteness (for propositional logics) turns out to be very helpful in this task. We do not restrict our attention to logical systems defined in the standard propositional language – such as classical or intuitionistic logic. We also take into consideration a family of modal logics which obey our criterium. We prove that **the density of truth exists for a large class of locally finite propositional logics** [3].
In [1], it is proved there that the implicational fragments of classical and intuitionistic logics are asymptotically equal if the densities of truth exist (for both logics).
Then we may strengthen the result from [1] as follows:

Theorem 1 *The densities $\mu(Cl_k^{\to})$ and $\mu(Int_k^{\to})$ of the implicational fragments of classical and intuitionistic logics exist and the following equality holds:* $\lim_{k\to\infty} \frac{\mu(Int_k^{\to})}{\mu(Cl_k^{\to})} = 1$

References

[1] Fournier H., Gardy D., Genitrini A., Zaionc M. *Classical and intuitionistic logic are asymptotically identical*, Lecture Notes in Computer Science 4646, pp. 177-193.

[2] Kostrzycka Z. *On the density of implicational parts of intuitionistic and classical logics*, Journal of Applied Non-Classical Logics, Vol. 13, Number 3, 2003, pp. 295-325.

[3] Kostrzycka Z. *On the Density of Truth of Locally Finite Logics*, Journal of Logic and Computation, Vol. 19 (6), (2009), pp. 1114–1125.

[4] Kostrzycka Z. and Zaionc M. *Statistics of intuitionistic versus classical logics*, Studia Logica, Vol. 76, Number 3, 2004, pp. 307-328.

[5] Moczurad M., Tyszkiewicz J., Zaionc M. *Statistical properties of simple types*, Mathematical Structures in Computer Science, vol 10, 2000, pp. 575-594.