

# On density of truth of infinite logic

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This paper concerns the problem of asymptotic similarity between classical and intuitionistic logics. It was introduced by M.Moczurad, J. Tyszkiewicz and M. Zaionc in the paper [7] and continued widely, see for example [2]-[6].

By infinite logic, we mean a logic, which in some language with a finite number of variables, has infinitely many classes of non-equivalent formulas. According to the above definition, the classical logic is finite, whereas the intuitionistic one - is not.

In the paper we solve the problem of existence of density of truth of infinite logic  $INT_p^{\rightarrow, \vee, \perp}$ . Its existence is a problem because the density of truth, as a function, is finitely additive only. It is not countably additive. So in the case of infinite family of sets  $\{A_i\}_{i \geq 0}$  we have only the inequality:

$$\mu \left( \bigcup_{i=0}^{\infty} A_i \right) \geq \sum_{i=0}^{\infty} \mu(A_i),$$

where  $\mu(A)$ - asymptotic density of  $A$ .

We prove the existence of density of truth of  $INT_p^{\rightarrow, \vee, \perp}$  and give a quite tight estimation of it. To do this, we apply analytic combinatorics and some algebraical methods as well.

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